

Math 436 (Spring 2020) - Homework 5

1. **Chapter 3:** 25
2. Let $X \times Y$ be the product space of topological spaces X and Y . If $A \subseteq X$ and $B \subseteq Y$, prove that
 - (a) $\overline{A \times B} = \overline{A} \times \overline{B}$;
 - (b) $(A \times B)^\circ = \overset{\circ}{A} \times \overset{\circ}{B}$.
3. If X and Y are discrete spaces, then the product space $X \times Y$ is discrete.
4. If X and Y are indiscrete spaces, then the product space $X \times Y$ is indiscrete.
5. Let (X, d_X) and (Y, d_Y) be two metric spaces. Consider the formula

$$D((x_1, y_1), (x_2, y_2)) := \sqrt{d_X(x_1, x_2)^2 + d_Y(y_1, y_2)^2}$$

for all (x_1, y_1) and (x_2, y_2) in $X \times Y$. Prove that D defines a metric on the set $X \times Y$.

6. (Bonus Question) Show that the topology on $X \times Y$ determined by the metric D given in Question #5 is exactly the product topology on $X \times Y$.